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The Tritone Paradox: Its Presence and Form of Distribution in a General Population

DIANA DEUTSCH & WILLIAM L. KUYPER
University of California, San Diego

YUVAL FISHER
Cornell University

The tritone paradox occurs when an ordered pair of tones is presented, with each tone consisting of a set of octave-related components, and the pitch classes of the tones separated by a half-octave. Such a pattern is heard as ascending in one key, but as descending in a different key. Further, the pattern in any one key is heard as ascending by some listeners but as descending by others. It was here found that this phenomenon occurs to a highly significant extent in a general population, and that it is distributed within the population in an orderly fashion. The findings also reveal a surprising ability within the general population to utilize absolute pitch.

THE pitch of a tone may be analyzed in terms of two components: The monotonic component of height defines its position on a continuum from high to low, and the circular component of pitch class defines its position within the octave (Babbitt, 1969, 1965; Bachem, 1948; Deutsch, 1969, 1973, 1982; Forte, 1973; Meyer, 1904, 1914; Revesz, 1913; Ruckmick, 1929; Shepard, 1964, 1965, 1982; Ward & Burns, 1982).

The distinction between pitch class and pitch height has long been considered self-evident by musicians. This is reflected, for example, in Western notational practice, in which notes of a given pitch class are assigned the same letter name regardless of the octave in which they are placed. The bidimensional nature of pitch has been shown to have certain perceptual and cognitive consequences. Generalization of conditioned response to tones standing in octave relation occurs both in human subjects (Humphreys, 1939; Demany and Armand, 1984) and in animals (Blackwell and Schlos-

Requests for reprints may be addressed to Diana Deutsch, Department of Psychology, University of California, San Diego, La Jolla, California 92093.

berg, 1943). Octave generalization effects are exhibited in short term memory for pitch (Deutsch, 1973). People with absolute pitch will sometimes make octave errors when assigning names to notes (Bachem, 1954; Baird, 1917; Lockhead and Byrd, 1981; Ward and Burns, 1982). Pairs of tones that stand in octave relation are judged as closely similar in a musical context (Krumhansl, 1979).

It has been proposed that the dimensions of pitch class and height can be accommodated in a single representation by depicting pitch as a helix which completes one full turn per octave. Tones which are an octave apart would thus be represented in close spatial proximity (see, for example, Figure 1). This proposal, which has a long history (see, for example, Drobisch, 1846) has been elaborated in detail by Shepard (1964, 1965, 1982).

The model of pitch as a geometrically regular helix assumes that the dimensions of height and pitch class are orthogonal. As pointed out by Shepard, this gives rise to the possibility that by minimizing the component of height, and so emphasizing the component of pitch class, all tones that are related by octaves could be mapped onto a single tone, which would then have a well-defined pitch class but an indeterminate height. Since the helix would in consequence be collapsed into a circle, judgments of relative height for such tones should be expected to be completely circular.

Shepard (1964) performed an experiment to test this prediction. He employed a set of tones, each of which consisted of 10 octave-related sinusoidal components, the amplitudes of which were determined by a fixed, bell-shaped spectral envelope. Such tones are heard as well-defined in terms of pitch class, but less well-defined in terms of height. The pitch classes of the tones were varied by shifting the sinusoidal components up and down in log frequency, keeping the location and shape of the envelope constant.

Subjects were presented with ordered pairs of such tones, and they judged whether each pair formed an ascending or a descending series.

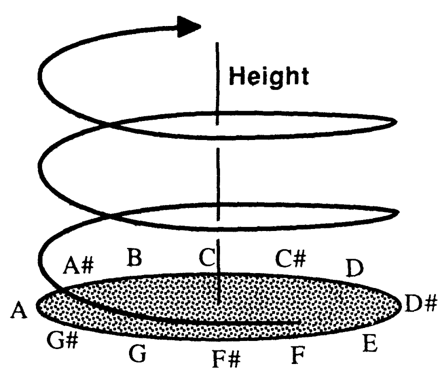

Fig. 1. The model of pitch as a geometrically regular helix. (Adapted from Shepard, 1965.)

When the tones within a pair were separated by one or two steps along the pitch class circle, judgments were almost entirely dependent on proximity. (For example, the series D♯–E was always heard as ascending, and the series G–F♯ was always heard as descending.) When the tones were separated by a greater distance along the pitch class circle, the tendency to follow by proximity lessened, and when the tones were separated by exactly a half-octave, ascending and descending judgments were made equally often.

This experiment demonstrated that for such tones, listeners will invoke proximity in making judgments of relative height [a phenomenon which has been confirmed by Burns (1981), Risset (1971), Schroeder (1986), Teranishi (1986), and Ueda and Ohgushi (1987)]. However, it left unexplored the situation where the cue of proximity is also removed. We may ask whether judgments of relative height would in this case be flat with respect to pitch class (as predicted from the assumption of orthogonality) or whether the two dimensions might be perceptually related. It should be noted that proximity has been found capable of overriding other cues in organizing pitch materials (Deutsch, 1975a, 1975b, 1985; Butler, 1979a, 1979b). Thus in Shepard's study other factors that might have produced differences in perceived height could have been masked. Further, in his experiment, the data were averaged across pitch classes, so that any influence of pitch class on perceived height would have been lost in the averaging process. The question of orthogonality of pitch class and height was thus left unresolved.

In an experiment to examine this issue, Deutsch (1986) employed tones which consisted of six octave-related sinusoids. A representation of the spectral composition of one such tone is given in Figure 2. Subjects were presented with ordered pairs of such tones, and they judged whether each pair formed an ascending or a descending series. The results were analyzed separately for each subject as a function of the pitch class of the first tone of the pair. The pitch classes of the tones within a pair were related by exactly a half-octave, so that they were diametrically opposed along the pitch class circle. (For example, C might be presented followed by F♯, or A followed by D♯, and so on.) Thus, proximity could not be used as a cue in making judgments of relative height.

Two paradoxical findings emerged from this study. First, the judgments of most subjects depended in a systematic fashion on the position of the tones along the pitch class circle, so that tones in one region of the circle were heard as higher and those in the opposite region as lower. (Thus, for a given subject, the pair D–G♯ might be heard as descending, and the pair G–C♯ as ascending.) As a second finding, the direction in which pitch class influenced judgments varied substantially from one subject to another. (So, for example, one subject might hear the pair E–A♯ as ascending, whereas another would hear the identical as descending.)

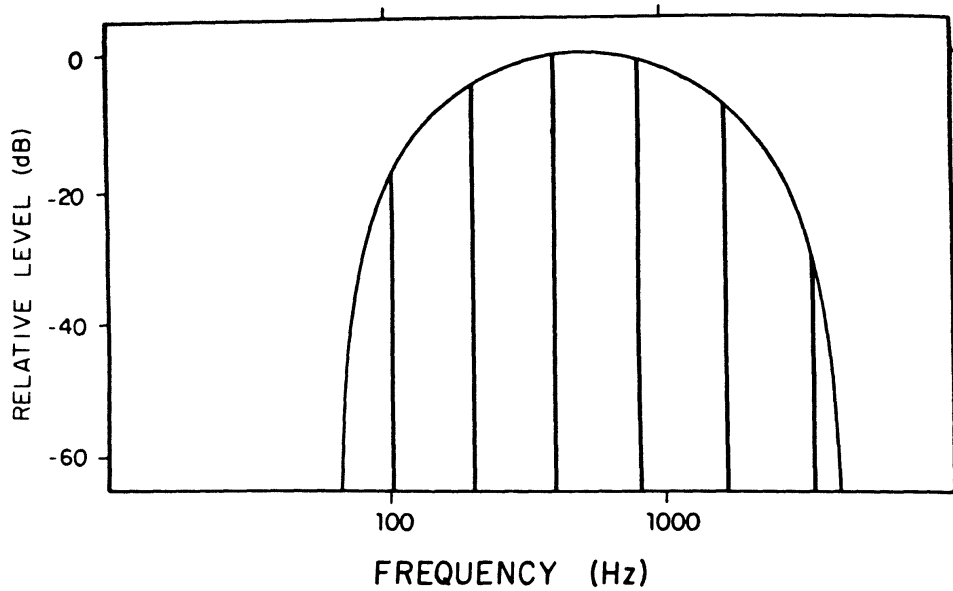

Fig. 2. Representation of the spectral composition of one of the tones employed in the study. In this case the spectral envelope is centered at C_5 (523 Hz) and the tone is of pitch class G♯.

Deutsch (1987) examined this phenomenon further, by exploring possible effects of variations in the position of the spectral envelope. In principle, such variations could affect judgments in two ways: first through differences in the overall heights of the tones, and second through differences in the relative amplitudes if their sinusoidal components. It was found that although some subjects showed an effect of overall height, and others showed an effect of relative amplitude, neither influence was necessarily present. Thus, the basic phenomenon cannot be attributed to either of the above factors.

The experiments of Deutsch (1986) and Deutsch (1987) explored judgments made by a few musically trained subjects. The question may then be raised of whether the tritone paradox is confined to a specialized group of listeners, or whether it might also occur in a general population. The present experiment was performed to examine this issue. A relatively large number of subjects were employed, and these were selected without regard for musical training. As will be shown, the influence of pitch class on perceived height was found to exist to a highly significant extent in this population. It was further found that the direction of the relationship between pitch class and perceived height varied across the subject population in a surprisingly orderly fashion.

Method

Stimulus Patterns

The tones employed in the experiment all consisted of six sinusoids which were related by octaves, and their amplitudes were determined by a fixed, bell-shaped spectral envelope (Figure 2). The general form of the equation describing the envelope is

$$A(f) = 0.5 - 0.5 \cos \left[\frac{2\pi}{\gamma} \log_\beta \left(\frac{f}{f_{min}} \right) \right],$$
$$f_{min} \leq f \leq \beta^\gamma f_{min},$$

where $A(f)$ is the relative amplitude of a sinusoid at frequency f Hz, β is the frequency ratio between adjacent sinusoids (thus, for example, for octave spacing, $\beta = 2$), γ is the number of β cycles that are spanned, and f_{min} is the minimum frequency for which the amplitude is non-zero. The maximum frequency for which the amplitude is non-zero is thus $\gamma\beta$ cycles above f_{min}. Throughout the experiment, the values $\beta = 2$ and $\gamma = 6$ were employed, so that the spectral envelope always spanned precisely 6 octaves from f_{min} to $64f_{min}$.

In order to control for possible effects based on the relative amplitudes of the sinusoidal components, tone pairs were generated with envelopes placed at four different positions along the spectrum, which were spaced at half-octave intervals. Figure 3 displays the envelope in these four positions, and it can be seen that their peaks stood at C_4 (262 Hz, $f_{min} = 32.7$ Hz), $F\sharp_4$ (370 Hz, $f_{min} = 46.2$ Hz), C_5 (523 Hz, $f_{min} = 65.4$ Hz) and $F\sharp_5$ (740 Hz, $f_{min} = 92.4$ Hz).

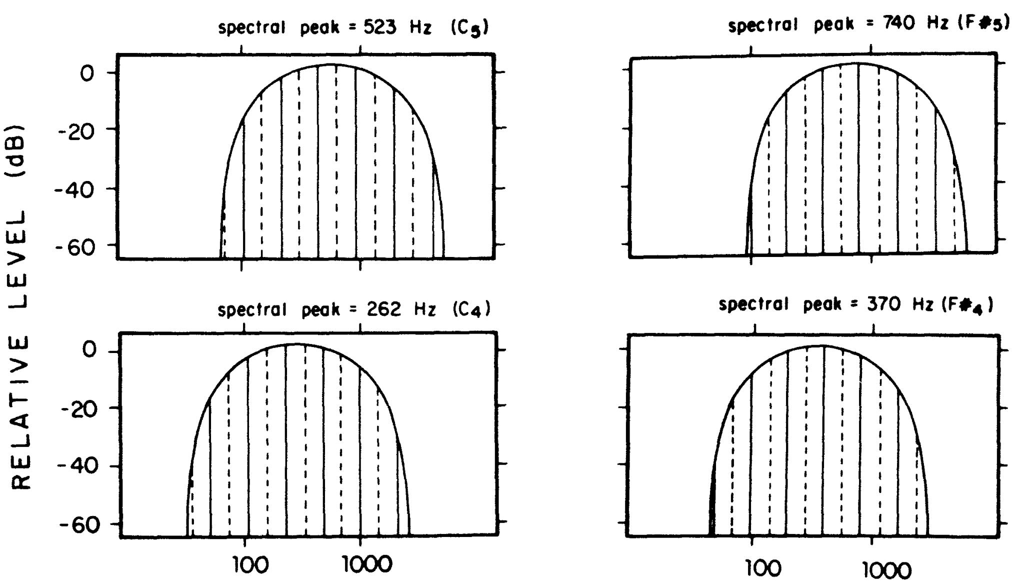

Fig. 3. Representation of the spectral composition of the tones comprising the D–G♯ pattern, generated under the four spectral envelopes used in the study. Dashed lines indicate tones of pitch class D, and solid lines tones of pitch class G♯. The two sets of spectra are here superimposed, but the tones were presented in succession.

It should be noted that the relative amplitudes of the sinusoidal components of tones at any given pitch class when generated under the envelopes centered at C_4 and C_5 were identical to those at the pitch class a half-octave removed when generated under the envelopes centered at F♯$_4$ and F♯$_5$. (For example, the sinusoidal components of the tones comprising the pattern D–G♯ when generated under envelopes centered at C_4 and C_5 were identical to those comprising the pattern G♯–D when generated under envelopes centered at F♯$_4$ and F♯$_5$.) Thus averaging results obtained under these different envelopes enabled the balancing out of any possible effects based on the relative amplitudes of the sinusoidal components of the tones.

Twelve ordered pairs of tones were generated under each of the four spectral envelopes. These corresponded to the pitch-class pairings C–F♯, C♯–G, D–G♯, D♯–A, E–A♯, F–B, F♯–C, G–C♯, G♯–D, A–D♯, A♯–E, and B–F. Forty-eight ordered tone pairs were thus produced altogether. These were presented in blocks of 12, each block composed of tones which were generated under one of the four envelopes and containing one example of each of the 12 pitch-class pairings. Within blocks, the 12 tone pairs were presented in any of four orders; these were random with the restriction that the same pitch classes did not occur in any two consecutive pairs. Sixteen blocks were thus created altogether, with the four within-block orderings employed once for each position of the spectral envelope.

Procedure

Subjects were tested in soundproof booths. On each trial, one of the tone pairs was presented, and subjects judged whether it formed an ascending or a descending series. All tones were 500 msec in duration, and there were no gaps between tones within a pair. Tone pairs within blocks were separated by 5-sec intertrial intervals, and blocks were separated by 1-min pauses, except for a 5-min break between the eighth and ninth blocks. A few practice trials were given at the beginning of the session.

Equipment

Tones were generated on a VAX 11/780 computer, interfaced with a DSC-200 Audio Data Conversion System, using the cmusic sound synthesis software (Moore, 1982). They were recorded and played back on a Sony PCM-F1 digital audio processor, the output of which was routed through a Crown amplifier and delivered to subjects binaurally through headphones (Grason-Stadler TDH-49) at a level of approximately 72 dB SPL.

Subjects

Twenty-nine undergraduates at the University of California, San Diego served as subjects in the experiment and were paid for their participation. All had normal hearing. They were selected without regard for musical training, on the basis of obtaining an errorless performance in a preliminary experiment in which they judged whether sinusoidal tone pairs which were related by a half-octave formed ascending or descending series. All subjects denied having absolute pitch, in the sense of being able to attach verbal labels to notes heard in isolation.

Results

For each subject, the percentage of judgments that a tone pair formed a descending series was plotted as a function of the pitch class of the first tone of the pair. As exemplified in the three graphs in Figure 4, such individual judgments were strongly influenced by the positions of the tones along the

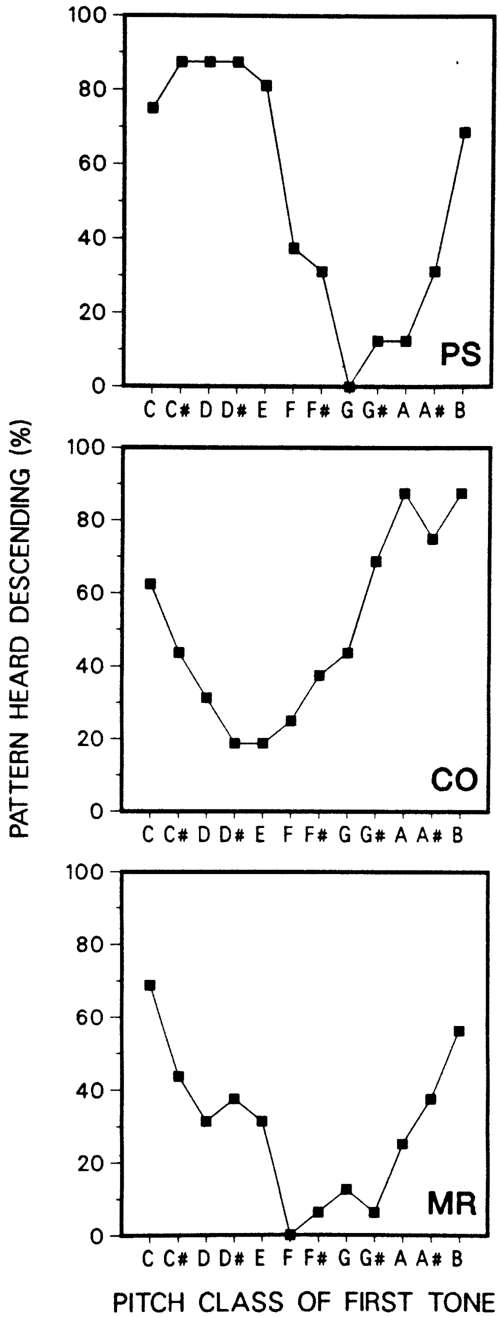

Fig. 4. Percentages of judgments that a tone pair formed a descending series, plotted as a function of the pitch class of the first tone of the pair. Results from three different subjects are here displayed, averaged across the four spectral envelopes.

pitch class circle. However, also as shown in Figure 4, the direction of this influence varied considerably between subjects.

An estimate was made of the prevalence of the effect in the subject population as a whole. First, it was determined for each individual subject's scores whether the pitch class circle could be bisected such that none of those in the upper half of the circle were lower than any of those in the lower half. The data of 22 of the 29 subjects were found to fulfill this criterion. Second, in order to obtain a baseline estimate of the probability of obtaining this result by chance, the proportion of random permutations of the scores which could be so characterized was determined by computer simulation. Averaged across all subjects, this was found to be .027 per subject. Thus the probability of obtaining the combined result by chance was vanishingly small. We may conclude that the phenomenon exists to a very highly significant extent in this general population.

We next enquired into the form of the relationship between pitch class and perceived height in the subject population as a whole. To this end, the orientation of the pitch class circle was normalized across subjects using the following procedure. For each subject, the pitch class circle was bisected so as to maximize the difference between the averaged scores within the two halves. The circle was then oriented so that the line of bisection was horizontal, and the data were retabulated, with the leftmost pitch class of the upper half of the circle taking the first position, its clockwise neighbor taking the second, and so on. The normalized data were then averaged. The resultant plot is shown in Figure 5 and reveals a remarkably orderly relationship between pitch class and perceived height.

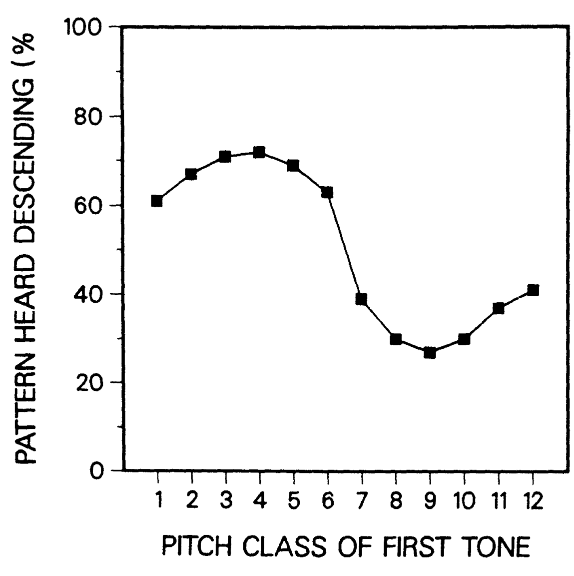

Fig. 5. Percentages of judgments that a tone pair formed a descending series, plotted as a function of the pitch class of the first tone of the pair. Results are averaged over all subjects (and across the four spectral envelopes), with the orientation of the pitch class circle normalized across subjects.

We next examined whether the phenomenon might be related to musical training. Subjects were designated as "trained" if they had had more than 2 years of musical training; otherwise they were designated as "untrained." Two analyses were performed. First, an estimate was obtained of the absolute size of the effect for each subject individually, by subtracting the averaged score for the lower half of the normalized circle from that for the upper half. It was found that of the 15 subjects who showed the larger difference on this measure, 7 were "untrained." Of the 14 subjects who showed the smaller difference, 7 were also "untrained." In fact, the two subjects who showed the largest difference on this measure had had no musical training whatsoever. For the second analysis, the proportions of "trained" and "untrained" subjects whose individual scores produced statistically significant results were determined, and again no significant difference between the two groups emerged. It appears clear from these analyses that the relationship between pitch class and perceived height cannot be attributed to musical training.

Finally, we examined whether the orientation of the pitch class circle varied haphazardly across subjects, or whether it was distributed within this population in an orderly fashion. To this end, the two pitch classes which stood at the peak of the normalized circle were tabulated, and the distribution of peak pitch classes within the population was determined. As shown in Figure 6, this distribution was found to be most orderly. Thus, for the most part, the pitch classes between B and D♯ were heard as higher, and those between F and A were heard as lower.

Discussion

The findings described here show that the perceived height of a tone can be systematically influenced by its position along the pitch class circle. This corroborates earlier findings on the tritone paradox by Deutsch (1986, 1987). The present findings show in addition that the phenomenon is not confined to a specialized group of listeners, but occurs to a highly significant extent in a general population.

The mechanism responsible for the tritone paradox is at present unknown. However, it should be emphasized that the employment of four spectral envelopes which were spaced at half-octave intervals controlled for any simple interpretations based on the relative amplitudes or loudnesses of the sinusoidal components of the tones; a point which was further documented in detail by Deutsch (1987). In this latter experiment, the phenomenon was also shown to be robust when subjects were tested over multiple sessions. A further study showed that the phenomenon was unaffected when the envelopes under which the tones were generated were stretched

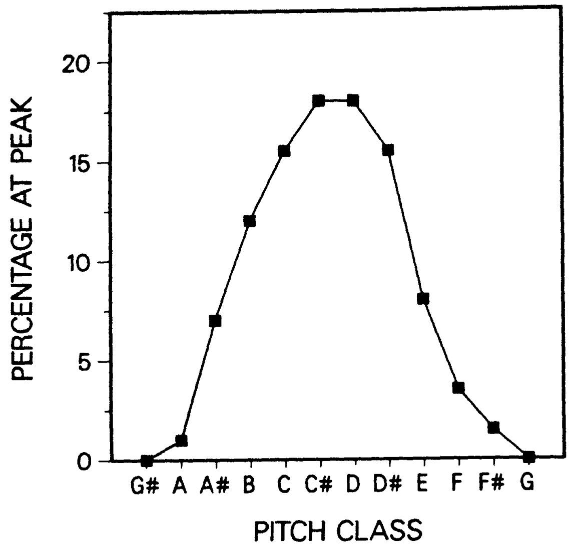

Fig. 6. Distribution of peak pitch classes within the subject population.

lightly, so that the sinusoidal components stood in a ratio of 2.01:1, with the result that the phase relationships between these components were constantly varying (Deutsch, in preparation).

A related influence of pitch class on perceived height was recently demonstrated by Deutsch, Moore, and Dolson (1984, 1986). Two-part patterns were here employed, which were also composed of octave-related complexes. It was found that subjects perceived such patterns quite differently depending on which key they were in, so that transposing the patterns resulted in a perceived interchange of voices. Further, as with the tritone paradox, striking differences between subjects emerged in the direction in which pitch class influenced perceived height.

Since these studies show that the perceived height of a tone is influenced by its position along the pitch class circle, they demonstrate that height and pitch class are not orthogonal dimensions, as assumed by the model of pitch as a geometrically regular helix (Figure 1). This point may be illustrated with reference to the earlier study on the tritone paradox by Deutsch (1986), in which a larger number of judgments were taken from each subject, and these were averaged over two sessions. According to the helical model, if the tones carried no height information, judgments of relative height should be flat with respect to pitch class. If, however, the tones did carry some height information, then the relationship between pitch class and perceived height would be expected to reverse repeatedly as the spectrum was traversed. However, Figure 7 shows, in a particularly clear case,

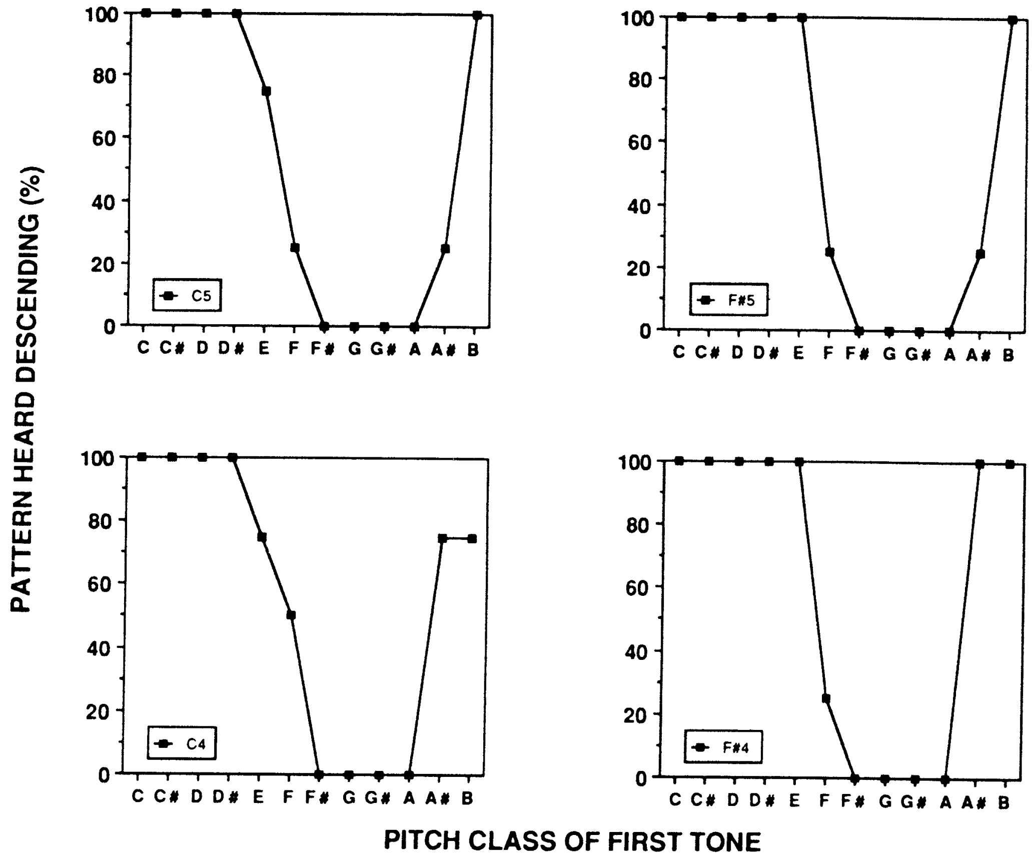

Fig. 7. Percentages of judgments that a tone pair formed a descending series, plotted as a function of the pitch class of the first tone of the pair. Results from a single subject are here displayed, for tones generated under each of the spectral envelopes used in the present study. Symbols in boxes indicate the peaks of the spectral envelopes. (Data from Deutsch, 1986.)

the judgments made by one of the subjects (F. T.) with tones generated under the four spectral envelopes employed in the present study. It can be seen that there was a pronounced and consistent relationship between pitch class and perceived height: Tones of pitch classes C, C♯, D, and D♯ were consistently heard as higher, and tones of pitch classes F♯, G, G♯, and A as lower, for all positions of the spectral envelope. These results cannot be accommodated on the model of pitch as a geometrically regular helix.

It will also be noted that the direction of the influence of pitch class on perceived height was distributed within the present subject population in an orderly fashion. As described above, pitch classes between B and D♯ were generally heard as higher, and those between F and A♯ as lower. It is at present unknown to what extent this particular distribution occurs in populations other than explored here; for example in other age groups and geographical regions. Similarly, we have no basis at present for determining whether the differences found between listeners in the direction of the relationship between pitch class and perceived height are innate or cultural in origin.

The findings described here also have implications for absolute pitch, which is generally assumed to exist only in rare individuals. Since judgments here were strongly influenced by pitch class, the subjects were indirectly employing absolute pitch to perform the task. This was true despite the fact that the subjects did not possess absolute pitch as conventionally defined. It appears, therefore, that absolute pitch occurs frequently in the general population, at least in this partial form. In relation to these findings, a series of experiments on key identification may be cited. Terhardt and Ward (1982) and Terhardt and Seewann (1983) found that musicians were able to judge whether or not a passage was performed in the correct key, even though most of the subjects denied possessing absolute pitch as conventionally defined.

It has yet to be determined to what extent the findings on the tritone paradox might generalize to tones of different spectral composition. However, informal work by one of us (D. D.) has shown that the phenomenon persists when the sinusoidal components within each tone complex are all replaced with sawtooth waves (and so with the partials constituting a harmonic series), or with square waves (and so with the odd-numbered partials of a harmonic series). It can be observed that the spectra of these more elaborate tone complexes are similar to those produced by a group of natural instruments playing simultaneously, with their fundamental frequencies standing in octave relation. Given such findings, we may speculate that analogous effects might be produced in certain orchestral contexts, especially when instruments play with multiple octave doublings. This intriguing possibility awaits further investigation.[1]

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